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Optimizing inventory decisions in a multi-stage supply chain under stochastic demands

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ABSTRACT

In this paper we consider the case of a three-stage non-serial supply chain system. This supply chain system involves suppliers, manufactures, and retailers. Production and inventory decisions are made at the suppliers and manufactures levels. The production rates for the suppliers and manufactures are assumed finite. In addition the demand at each end retailer is assumed to be stochastic. The problem is to coordinate production and inventory decisions across the supply chain so that the total cost of the system is minimized. For this purpose, we develop a model to deal with different inventory coordination mechanisms between the chain members. We present a numerical example for illustrative purposes.

1. Introduction

Supply chain management can be defined as a set of approaches utilized to efficiently integrate suppliers, manufactures, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide cost while satisfying service level requirements [20].

In recent years numerous articles in supply chain modeling have addressed the issue of inventory coordination. Banerjee [1] introduced the concept of joint economic lot sizing problem (JELS). He considered the case of a single vendor and a single purchaser under the assumption of deterministic demand and lot for lot policy. He analyzed the effects of each party's optimal lot size on the other in case of independent optimization and developed a JELS model that focused on the joint total relevant cost (JTRC). Goyal and Szendrovits [9] presented a constant lot size model where the lot is produced through a fixed sequence of manufacturing stages, with a single setup and without interruption at each stage. Transportation of partial lots, called batches, is allowed between stages. This model mainly, relaxes the constraint that batches must be of equal size at any particular stage. Goyal [6] provided a more general model for the case of single vendor single buyer through relaxing the lot-for-lot policy. He assumed that the whole production lot should be produced before shipments take place. He showed that his model provides a lower or equal total joint relevant cost compared to [1].

Goyal and Gupta [8] extensively reviewed the literature which deals with the interaction between a buyer and vendor. They classified the literature dealing with the integrated models into four main classes. The first class represents models which deal with joint economic lot sizing policies. The second class characterizes models which deal with the coordination of inventory by simultaneously determining the order quantity for the buyer and the vendor. The third class is a group of models which deal with integrated problem but do not determine simultaneously the order quantity of the buyer and the vendor. The last class represents models which deal with buyer vendor coordination subject to marketing considerations. Lu [15] developed a one-vender multi-buyer integrated inventory model with the objective of minimizing the vender's total

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annual cost subject to the maximum cost that the buyer may be prepared to incur. They have found the optimal solution for the single vendor single buyer case under the stated assumptions and presented a heuristic approach for the one-vendor multi-buyer case. Goyal [7] in his short paper revisited the single vendor single buyer where he relaxed the constraint of equal sized shipments of Goyal [6] and suggested that the shipment size should grow geometrically. Lu [15] considered a single manufacturer single supplier supply chain where the manufacturer orders its raw materials from its supplier, the converts the raw materials into finished goods, and finally delivers the finished goods to its customers. He proposed an integrated inventory control model that comprises of integrated vendor–buyer (IVB) and integrated procurement-production (IPP) systems.

~~Hooke and Jeeves~~ [12] extended the idea of producing a single product in a multi-stage serial production system with equal and unequal sized batch shipments between stages. ~~Moutaz~~ Khouja [17] considered the case a three-stage non-serial supply chain and developed the model to deal with three inventory coordination mechanisms between the chain members. Bendaya and Al-Nassar [3] relaxed the assumption of ~~Moutaz~~ Khouja [17] regarding the completion of the whole production lot before making shipments out of it and assumed that equal sized shipments take place as soon as they are produced and there is no need to wait until a whole lot is produced.

Cárdenas-Barrón [4] formulated and solved an n-stage-multi-customer supply chain inventory model where there is a company that can supply products to several customers. The production and demand rates were assumed constant and known. This model was formulated for the simplest inventory coordination mechanism which is referred to as the same cycle time for all companies in the supply chain. It was concluded that it is possible to use an algebraic approach to optimize the supply chain model without the use of differential calculus.

Han [10] established a strategic resource allocation model to capture and encapsulate the complexity of the modern global supply chain management problem. He constructed a mathematical model to describe the stochastic multiple-period two-echelon inventory with the many-to-many demand-supplier network problem. He applied Genetic algorithm (GA) to derive optimal solutions through a two-stage optimization process. His model simultaneously constitutes the inventory control and transportation parameters as well as price uncertainty factors.

Chung and Wee [5] considered an integrated three-stage inventory system with backorders. They formulated the problem to derive the replenishment policies with four-decision-variables algebraically. Long et al. [14] studied a supply chain model in which a single supplier sells a single product to a single retailer who faces the newsvendor problem. The retailer is loss averse. The results showed that the optimal production quantity with decentralized decision making with a wholesale price contract is less than that with centralized decision making. The supply chain can achieve channel coordination with buy back and target rebate contracts. With buy back contracts, the supply chain system profits can be allocated arbitrarily between the supplier and retailer. A new kind of contract, the incremental buy back contract, gives similar results as with the buy back contract. They analyzed advantages and drawbacks of these three types of contracts via numerical examples.

Barnes-Schuster et al. [2] studied a system composed of a supplier and buyer(s). They assumed that the buyer faces random demand with a known distribution function. The supplier faces a known production lead time. Their main objective was to determine the optimal delivery lead time and the resulting location of the system inventory. For a system with a single supplier and as single buyer, they showed that system inventory should not be split between a buyer and supplier. They also derived the conditions indicating when the supplier or buyer(s) should keep the system inventory, based on systems parameters of shortage and holding costs, production lead times, and standard deviations of demand distributions. Man-Yi and Xiao-Wo [16] studied how to evaluate the safety stock of node enterprise given desired product availability when market demand of the node enterprise in supply chain is described by Gauss fuzzy variable. They discussed the impact of required product availability and demand uncertainty on safety stock, compared the correlative issues with stochastic demand, and got some useful results. Rau and OuYang [19] presented an integrated production–inventory policy under a finite planning horizon and a linear trend in demand. They assumed that the vendor makes a single product and supplies it to a buyer with a non-periodic and just-in-time (JIT) replenishment policy in a supply chain environment. They first, developed a mathematical model and proved that it has the optimal solution. Then, they described an explicit solution procedure for obtaining the optimal solution and they provided two numerical examples to illustrate both increasing and decreasing demands in the proposed model. Nagarajan and Sošić [18] surveyed some applications of cooperative game theory to supply chain management. They first, described the construction of the set of feasible outcomes in commonly seen supply chain models, and then used cooperative bargaining models to find allocations of the profit pie between supply chain partners. They analyzed and surveyed several models. Then they discussed the issue of coalition formation among supply chain partners. They presented an exhaustive survey of commonly used stability concepts.

As mentioned earlier numerous articles in supply chain modeling have been written in response to the global competition. However, most of the developed supply chain inventory models deal with two-stage supply chains. Even when multi-stage supply chains are considered, most of the developed models are based on restrictive assumptions such as of the deterministic demand. Therefore, there is a need to analyze models that relax the usual assumptions to allow for a more realistic analysis of the supply chain inventory coordination. In this paper, we extend [17] by relaxing the deterministic demand and assume that the end retailers face stochastic demand.

The remainder of this paper is organized as follows. The next section presents problem Definition, Notations and assumptions. Section 3 describes the development of the model. A numerical example is presented in Section 4. Finally, Section 5 contains some concluding remarks.

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2. Problem definition

Consider the case of a three-stage supply chain where a firm can supply many customers. This supply chain system involves suppliers, manufactures and retailers. Production and inventory decisions are made the at suppliers and manufactures levels. The production rates for the suppliers and manufactures are assumed finite. In addition the demand for each firm is assumed to be stochastic. The problem is to coordinate production and inventory decisions across the supply chain so that the total cost of the system is minimized.

The following notations are used in developing the models:

T	Basic cycle time, cycle time at the end retailer
~~T_i~~	~~Cycle time at the stage i~~
A_i	Setup cost at stage i
K_i	Integer multiplier at stage i
h_i	Inventory holding cost at stage i
n_i	Number of firms at stage i
D_{ij}	The mean demand rate of firm j at stage i
P_{ij}	Production rate of firm j at stage i
x	A random variable describing the demand at retailer j
$f_{i,j}(x,T)$	The continuous probability density of the customer demand received at retailer j in stage i during the period T
π	The shortage penalty per unit short

3. Model development

In this work we deal with two coordination mechanisms.The first is the simple equal cycle time coordination mechanism where the same cycle time is adopted at all stages. The second coordination mechanism is the integers multipliers in which firms at the same stage of the supply chain use the same cycle time and the cycle time at each stage is an integer multiplier of the cycle time used at the adjacent downstream stage.

The formulation of the three multi-stage, multi customers, non-serial supply chain according to the two coordination mechanism is presented in the following two subsections.

3.1. Equal cycle time coordination

Q2 Let i = 1,2, and 3 denote the stage index in the supply chain. And let = 1,2,..., J_i be an index denoting firms within each stage. As we can see from Fig. 1 the expected total cost per unit time for a downstream retailer can be approximated as

$$TC_{3,j} = h_3 \int_0^{TD_{3,j}} \left(TD_{3,j} - \frac{x}{2}\right) f_{3,j}(x,T)\mathrm{d}x + h_3 \int_{TD_{3,j}}^{\infty} \frac{(TD)^2}{2x} f_{3,j}(x,T)\mathrm{d}x + \pi \int_{TD}^{\infty} \frac{1}{2x}(x - TD)^2 f_{3,j}(x,T)\mathrm{d}x + \frac{A_3}{T}, \tag{1}$$

where $f_{3j}(x,T)$ is the continuous probability density of the demand at the jth retailer, during the period time of length$T$. The first and second terms in Eq. (1) represents the average carrying cost at an end retailer, while the third term is the average shortage cost. The last term is the replenishment cost, which occurs in every period of length T when the product is ordered.

The annual total cost for a manufacturer at the second stage is made up of two parts: the first is cost of carrying raw material as they are transformed to final products; and the second is the cost of holding finished goods. This occurs only during the production portion of the cycle. During the non-production portion of the equal cycle the inventory drops to zero because the whole amount produced is immediately shipped to the retailer stage. During the production portion, the average raw material and finished products inventory is $TD_{2,j}/2$. Since the production rate is $P_{2,j}$, the per unit time average raw material and finished goods holding costs are $h_1 TD_{2,j}^2/2P_{2,j}$ and $h_2 TD_{2,j}^2/2P_{2,j}$, see [17]. Hence the expected total cost for a firm at stage 2 (i.e. manufacturer) is

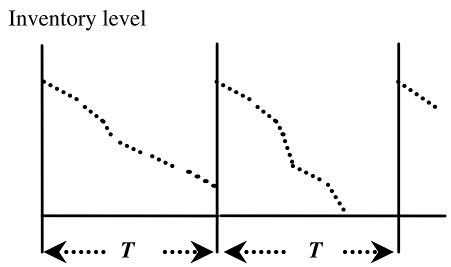

Fig. 1. Inventory level at a retailer.

$$TC_{2,j} = \frac{TD_{2,j}^2}{2P_{2,j}}(h_1 + h_2) + \frac{A_2}{T}. \quad (2)$$

Similarly the expected total cost for a firm at stage 1 (i.e. supplier) is

$$TC_{1,j} = \frac{TD_{1,j}^2}{2P_{1,j}}(h_0 + h_1) + \frac{A_1}{T}. \quad (3)$$

3.2. *Integers multipliers coordination*

For the integers multipliers coordination mechanism, firms at the same stage of the supply chain use the same cycle time and the cycle time at each stage is an integer multiplier of the cycle time used at the adjacent downstream stage. In this case, the cycle time of a retailer is T and that of a manufacturer is K_2T where K_2 is a positive integer. The cycle time of a supplier is a multiple of that of the manufacturer and is equal to K_1K_2T, see [17].

The total cost per unit time for retailer j is given by the same expression (1) in the previous section. The total cost for a firm at stage 2 (i.e. manufacturer) is given by

$$TC_{2,j} = \frac{K_2TD_{2,j}^2h_1}{2P_{2,j}} + \frac{A_2}{K_2T} + \frac{TD_{2,j}}{2}(K_2(1 + D_{2,j}/P_{2,j}) - 1)h_2. \quad (5)$$

The cost of carrying raw material as it is being converted into finished goods is given by the first term on the right-hand side of Eq. (5). However, the second term is made up of two parts. The first part is $K_2TD_{2,j}^2h_2/2P_{2,j}$ which is the per unit time holding cost of the finished goods for the non-production portion of the cycle and similar to the cost in the equal cycle mechanism. The second part is $TD_{2,j}(k_2 - 1)h_2/2$ which represents the per unit time holding cost for the non-production portion of the cycle, see [17]. Similarly the total cost for a firm at stage 1 (i.e. supplier) is

$$TC_{1,j} = \frac{K_1K_2TD_{1,j}^2h_0}{2P_{1,j}} + \frac{K_2TD_{1,j}^2}{2}(K_1(1 + D_{1,j}/P_{1,j}) - 1)h_1 + \frac{A_1}{K_1K_2}. \quad (6)$$

4. Numerical analysis

In this section, we consider an example of a three-stage supply chain having one supplier, three manufacturers, and seven retailers. The relevant data is shown in Table 1. This is similar to the example used in [17] except that the demand is stochastic in our case. The demand at the end retailers is assumed to follow normal distributions. In addition to the data in Table 1, it is assumed that the shortage cost per unit time per unit short is 0.08. A direct search program based on Hooke and Jeeves as described in [11] is developed to find the optimal solution. Under the equal time mechanism, the optimal cycle time is 0.0697 years and the expected total cost TC = $61647.2365 per year. Under the integer multipliers mechanism, the basic cycle time at the retailers is 0.062 years. The integer multiplier for the manufactures is $K_2 = 1$, and hence the cycle time at this stage equals the basic cycle time. However the integer multiplier for the supplier is $K_1 = 2$ and consequently the cycle time at this stage is 0.124 years and the total cost drops by 7159.5726 to TC = $54487.6639 per year.

Table 1
Example data

j	Order cost	Holding cost	Demand	
			Mean	Variance
Retailers				
1	50	5.0	10,000	500
2	50	5.0	20,000	500
3	50	5.0	40,000	500
4	50	5.0	12,000	500
5	50	5.0	24,000	500
6	50	5.0	9,000	500
7	50	5.0	18,000	500
Manufacturers				
j	Set up cost	Holding cost	Production rate	
1	200	(0.8, 2.0)	140,000	
2	200	(0.8, 2.0)	108,000	
3	200	(0.8, 2.0)	108,000	
Supplier				
1	800	(0.08, 0.8)	399,000	

Table 2
Sensitivity analysis when (A_1,A_2) are decreased

(A_1,A_2) decrease	25%		50%		75%	
	T^*	ETC^*	T^*	ETC^*	T^*	ETC^*
Equal time cycle	0.0617	56317.16	0.0526	50109.88	0.0419	42781.04
Integer multipliers	0.0554	50226.47	0.0479	45383.51	0.047	39614.13
Saving		6090.695		4726.365		3166.91

Table 3
Sensitivity analysis when (h_0, h_1, and h_2) are increased

(h_0, h_1, and h_2) multipliers	1.25		1.50		2.00	
	T^*	ETC^*	T^*	ETC^*	T^*	ETC^*
Equal time cycle	0.0673	63269.21	0.0652	64838.1	0.0615	67835.92
Integer multipliers	0.0602	55682.43	0.0585	56843.21	0.0754	58156.35
Saving		7586.779		7994.891		9679.57

We perform some sensitivity analysis on saving from using integer multipliers mechanism over the equal time cycle mechanism. We first decrease the current values of the setup cost at the suppliers and manufacturers stages by 25%, 50% and finally 75%. The results are given in Table 2. We then increase the values of (h_0, h_1, and h_2) by using multipliers of (1.25, 1.50 and 2.00) of the original values. The results are presented in Table 3.

5. Conclusion

In this paper we consider the case of a three-stage supply chain. This supply chain system involves suppliers, manufactures, and retailers. Production and inventory decisions are made the suppliers and manufactures levels. The production rates for the suppliers and manufactures are assumed finite. In addition the demand for each firm is assumed to be stochastic. We formulated a model to deal with two inventory coordination mechanisms between the chain members. The numerical results show that the integer multipliers coordination mechanism has lower cost than the equal cycle time coordination mechanism.

6. Uncited reference

Q1 [13].

References

[1] A. Banerjee, A joint economic-lot-size model for purchaser and vendor, Decision Sciences 17 (1986) 292–311.
[2] D. Barnes-Schuster, Y. Bassok, R. Anupindi, Optimizing delivery lead time/inventory placement in a two-stage production/distribution system, European Journal of Operational Research 174 (2006) 1664–1684.
[3] M. Bendaya, A. Al-Nassar, Integrated multi-stage multi-customer supply chain. Working paper, ~~Systems Engineering Department, King Fahd University of Petroleum and Minerals, 2005,~~
[4] L.E. Cárdenas-Barrón, Optimizing inventory decisions in a multi-stage multi-customer supply chain: a note, Transportation Research Part E: Logistics and Transportation Review, ~~in press,~~ Q3
[5] C.J. Chung, H.M. Wee, Optimizing the economic lot size of a three-stage supply chain with backordering derived without derivatives, European Journal of Operational Research 183 (2007) 933–943.
[6] S.K. Goyal, A joint economic-lot-size model for a purchaser and vendor: a comment, Decision Sciences 19 (1988) 236–241.
[7] S.K. Goyal, A one-vendor multi-buyer integrated inventory model: a comment, European Journal of Operational Research 82 (1995) 209–210.
[8] S.K. Goyal, Y.P. Gupta, Integrated inventory models: the buyer–vendor coordination, European Journal of Operational Research 41 (1989) 261–269.
[9] S.K. Goyal, A.Z. Szendrovits, A constant lot size model with equal and unequal sized batch shipments between production stages, Engineering Costs and Production Economics 10 (1986) 203–210.
[10] C. Han, Stochastic modeling of a two-echelon multiple sourcing supply chain system with genetic algorithm, Journal of Manufacturing Technology Management 16 (1) (2005) 87–108.
[11] R. Hooke, T.A. Jeeves, Direct search solution of numerical and statistical problems, Journal of the Association for Computing Machinery (1961) 212–229.
[12] M.A. Houqe, S.K. Goyal, An optimal policy for a single-vendor single-buyer integrated production–inventory system with capacity constraint of the transport equipment, International Journal of Production Economics 65 (2000) 305–315.
~~[13] Young Hae Lee, Min Kwan Cho, Yun Bae Kim, A discrete-continuous combined modeling approach for supply chain simulation, Simulation 78 (5 SPEC) (2002) 321–329.~~
[14] Z. Long, S. Shiji, W. Cheng, Supply chain coordination of loss-averse newsvendor with contract, Tsinghua Science and Technology 10 (2) (2005) 133–140.
[15] L. Lu, A one-vendor multi-buyer integrated inventory model, European Journal of Operational Research 81 (1995) 312–323.
[16] T. Man-Yi, T. Xiao-Wo, The further study of safety stock under uncertain environment, Fuzzy Optimization and Decision Making 5 (2) (2006) 193–202.
[17] Khouja Moutaz, Optimizing inventory decisions in a multi-stage multi-customer supply chain, Transportation Research Part E: Logistics and Transportation Review, Exeter (2003) 193–208.

Please cite this article in press as: M.E. Seliaman, A.R. Ahmad, Optimizing inventory decisions in a multi-stage supply chain under ..., Appl. Math. Comput. (2008), doi:10.1016/j.amc.2008.05.057

[18] M. Nagarajan, G. Sošić, Game-theoretic analysis of cooperation among supply chain agents: review and extensions, European Journal of Operational Research 187 (3) (2008) 719–745.
[19] H. Rau, B.C. OuYang, An optimal batch size for integrated production–inventory policy in a supply chain, European Journal of Operational Research 185 (2) (2008) 619–634.
[20] D. Simch-Levi, P. Kaminsky, E. Simch-Levi, Designing and Managing the Supply Chain: Concepts, Strategies, and Case Studies, second ed., McGraw-Hill, 2003.

Please cite this article in press as: M.E. Seliaman, A.R. Ahmad, Optimizing inventory decisions in a multi-stage supply chain under ..., Appl. Math. Comput. (2008), doi:10.1016/j.amc.2008.05.057